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INELASTIC SEISMIC TORSIONAL BEHAVIOUR OF ELEVATED TANKS

S. C. Dutta[†]

Department of Applied Mechanics, Bengal Engineering College (Deemed University), Howrah 711103, *India. E-mail: scdutta@mailcity.com*

S. K. Jain

Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, *India.*

AND

C. V. R. Murty

Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, *India*

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Torsional failure of elevated tanks has occurred in past earthquakes. The overall axisymmetric structural geometry and mass distribution of such structures may leave only a small accidental eccentricity between centre of stiffness and centre of mass. Such a small accidental eccentricity is not expected to cause a torsional failure. This paper studies the possibility of amplified torsional behaviour of elevated water tanks due to such small accidental eccentricity in inelastic range through detailed case studies; using two simple idealized systems with two coupled lateral–torsional degrees of freedom and, strength-deteriorating and elasto-plastic hysteresis models. The systems are capable of retaining the characteristics of two extreme categories of water tanks namely, (a) tanks on staging with less number of columns and panels and (b) tanks on staging with large number of columns and panels. The study shows that the presence of a small eccentricity may lead to localized unsymmetrical yielding in some of the reinforced concrete staging elements. This may lead to progressive strength deterioration through successive yieldings in same elements under cyclic loading during earthquakes. Such localized strength drop may increasingly develop large strength eccentricity resulting in large localized inelastic displacement and ductility demand, leading to failure. These observations are also verified for a real-life example elevated tank. The tanks supported on staging with fewer columns and panels are found to have greater torsional vulnerability. The tanks located near a fault are found to be vulnerable under near-fault pulses with a large duration compared to the lateral period of tank.

1. INTRODUCTION

Torsional failure of some reinforced concrete as well as steel elevated water tanks has occurred in past earthquakes [1–3]. The latest failure of this kind was the torsional failure of a reinforced concrete elevated water tank during 1993 Killari, India, earthquake [3]. In this case, the tank container vertically collapsed burying the six supporting columns directly underneath the bottom slab of the container. This vertical collapse and the evidence of

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[†] Formerly Graduate Student, Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India.

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a circumferential displacement of about 0·5 m suggest that torsional vibrations may have been the primary cause of failure. But, elevated water tanks with their broadly axisymmetric geometry and mass distribution, do not appear to be prone to torsion since, centre of resistance (CR) and centre of mass (CM) tend to coincide. However, small eccentricity between CR and CM may arise due to accidental reasons. For instance, a geometrical imperfection or non-uniformity in construction in the columns could slightly move the CR. Sloshing of the water mass also may cause a shift in the CM at a given instant of time. Asymmetric placements of ladders, stairs and pipelines may also give rise to a small eccentricity. Such accidental eccentricity may cause yielding asymmetrically in some of the staging elements and continued strength deterioration of the same elements in each new inelastic excursion. This will initiate coupled lateral-torsional inelastic behaviour in elevated tanks.

Numerous studies on the non-linear behaviour of eccentric systems under lateral–torsional coupling have been reported in the literature [4,5]. Most of them have studied the response under recorded earthquake ground motions or associated spectra, and of systems without strength deterioration but under cyclic loading. The amplification in torsional response due to lateral–torsional coupling is reported to be absent in small-eccentricity-yielding systems with elasto-plastic characteristics for resisting elements [6]. This is attributed to the considerable de-tuning of the uncoupled lateral and torsional natural periods on yielding. In fact, two earlier studies [7, 8] concluded that the peak ductility demand in small-eccentricity systems is similar to that in the corresponding symmetric systems. Also, it is reported that inclusion of the stiffness degrading characteristics in the load-resisting element does not appreciably change the peak ductility demand in comparison with that obtained without including the same [6]. But, the effect of strength deterioration was not recognized in these studies.

Inelastic torsional behaviour of reinforced concrete elevated tanks supported on frame-type staging (Figure 1) is studied in this paper to observe the effect of strength deterioration of the reinforced concrete members of the tank staging under cyclic loading. The asymmetrically large elastic displacement at one edge of the staging may cause localized yielding and considerable strength drop in successive inelastic excursion, even due to a small triggering eccentricity [9]. The present study is an effort to see whether the large

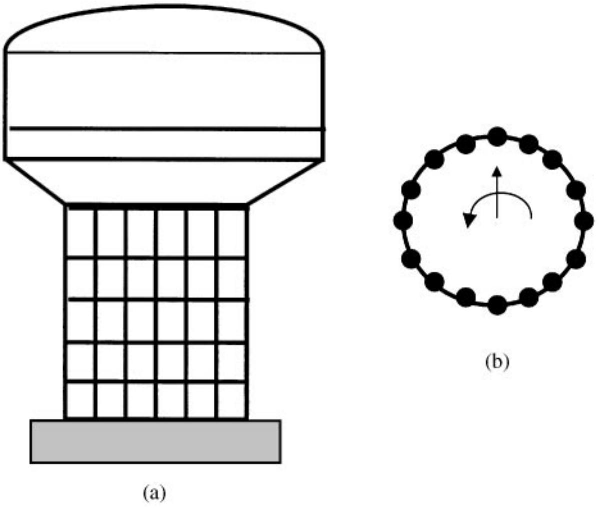
(a) (b)

Figure 1. Two-degress-of-freedom idealization of an elevated water tank on frame staging. (a) Elevation, (b) Staging plan

strength eccentricity from such progressive effect of torsion may create a very high localized displacement and ductility demand. The behaviour of elevated tanks under long duration near-fault pulses are also studied as it is pointed out that such near-fault pulses may be crucial for structures [10–12].

2. MODELLING OF ELEVATED WATER TANKS

2.1. IDEALIZED SYSTEMS

Generally, elevated water tanks have two types of modes of lateral vibration namely, the impulsive and the sloshing modes of vibration. Similarly, impulsive and sloshing modes of vibration also exist under torsional motion. The natural periods of the sloshing modes of vibration of elevated water tanks are usually quite large in comparison with both the impulsive natural periods and the periods of the pulses in earthquake ground motions. This is evident from a few example tank problems solved in the literature [13]. The impulsive mode of vibration strongly dominates the dynamic behaviour of elevated water tanks due to the participation of the whole structural mass and a major part of the water mass. Hence, it is considered that the coupling between impulsive modes of vibration in translation and torsion will primarily generate the torsional vibration in elevated water tanks during earthquakes. Accordingly, only impulsive modes of vibration, in translation and torsion, are considered in this study. So, the structure is modelled as a single-storey system with two degrees of freedom, namely the lateral translation and the rotation of the CM in horizontal plane (Figure 1).

Elevated water tanks have resisting elements (columns) of circular frame staging placed near the perimeter of the tank (Figure 2(a) and 2(c)). The lateral stiffness of these elements are represented by the load-resisting elements near the edges of the idealized systems as shown in the plan in Figure 2(b) and 2(d), respectively. The load-resisting elements are often referred as element for simplicity in the rest of the study. These elements in the idealized systems are assumed to have only in-plane stiffness as indicated in the figure and no out-of-plane stiffness. So, the idealized models contain a rigid-floor diaphragm, representing

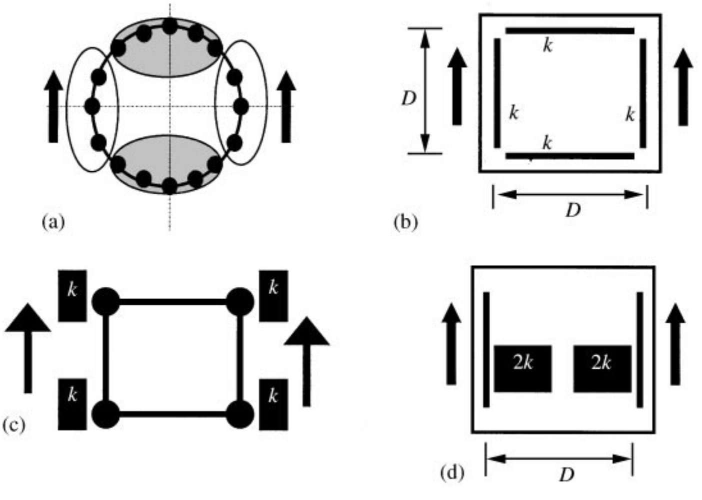

Figure 2. Plan of stagings and corresponding idealized single storey structural systems: (a) actual staging with many columns; (b) idealized four-element system ($K_x = 2k$, $K_\theta = kD^2$); (c) actual staging four columns; (d) idealized two-element with system ($K_x = 4k$, $K_\theta = kD^2$).

the comparatively rigid container of the tank, supported by four and two lateral load-resisting elements, and referred as four-element and two-element systems respectively.

The plan of a staging with many columns and the corresponding idealized four-element system are shown in Figure 2(a) and 2(b), respectively. Four separate groups of columns are identified and marked in the figure. The two groups of columns marked by unshaded ellipses have the circumferential beams approximately spanning along the direction of ground motion (shown by arrows in the figure). These columns mainly contribute lateral stiffness in the direction of ground motion, and can be adequately represented by two parallel load-resisting elements along the direction of ground motion, a distance D (i.e., physically representing the diameter of the staging) apart. The two groups of columns marked by shaded ellipses mainly contribute to the stiffness of the staging in the direction perpendicular to that of ground motion. So, these columns do not contribute lateral stiffness in the direction of ground motion; however, these columns do contribute torsional stiffness. In the four-element idealized system, these groups of columns are represented by the two elements located at a distance D from each other along the direction perpendicular to that of ground motion (Figure 2(b)). In stagings with four columns, the orientation of circumferential beams is such that all four columns equally participate in resisting the lateral force as well as torsional moment approximately with same stiffness, say, k (Figure 2(c)). Such stagings are represented by the two-element system shown in Figure 2(d) wherein the two load-resisting elements (each with stiffness $2k$) are at a distance D apart where D represents the diameter of the staging, i.e., the distance between two diametrically opposite columns. These two elements equally participate in resisting lateral force as well as torsional moment. Further details of the figure and model are available elsewhere [16]. Now, these idealized models are compared with actual elevated water tanks regarding the ratio of torsional and lateral stiffness and the ratio of torsional and lateral strength as described below. These are two important parameters which may regulate lateral–torsional coupled behaviour in elastic as well as post-elastic range.

A closed-form expression was derived for lateral stiffness, (K_x), of frame staging shown in Figure 1 by the method outlined in the literature [14] and assuming equal heights for all panels. The closed-form expression for torsional stiffness (K_θ) for the same was also derived similarly [15, 16]. Subsequently, the expression for ratio of torsional and lateral stiffnesses was obtained as follows:

$$\frac{K_\theta}{K_x} = \frac{D^2}{4}\left[\frac{0{\cdot}0025N_p(4N_p^2 - 1) + N_p + 2(N_p - 1)K_r}{N_p + (N_p - 1)K_r/\cos^2(\pi/N_c)}\right], \tag{1a}$$

where N_p, N_c and K_r are number of panels, number of columns and the ratio of flexural stiffness of columns and beams respectively. The variation of non-dimensionalized ratio $K_\theta/(K_xD^2)$ and the variation of the ratio (τ) of uncoupled torsional period (T_b) and uncoupled lateral period (T_x) for such tanks are reported elsewhere [15, 17]. The above expression shows that for stagings with less number of columns and panels, the non-dimensionalized stiffness ratio $K_\theta/(K_xD^2)$ approaches 0·25. For instance, this ratio is 0·255 when N_p, N_c and K_r, are all 4. For stagings with considerably large number of columns and panels, the ratio approaches 0·5 (e.g., if N_p, N_c and K_r are 8, 20 and 4, respectively, the ratio becomes 0·47). If D is the distance between two extreme end elements of the idealized systems in either direction, the ratio of torsional stiffness K_θ and lateral stiffness K_x may be expressed in non-dimensional form as

$$\frac{K_\theta}{K_xD^2} = \begin{cases} 0{\cdot}25 & \text{for a two-element system,} \\ 0{\cdot}50 & \text{for a four-element system.} \end{cases} \tag{1b}$$

Hence, the four-element systems reflect the extreme stiffness ratio of stagings with large number of columns, and panels. On the other hand, the two-element systems have stiffness ratio closer to that of stagings with four columns and small number of panels. An increase in the number of columns considerably increases the stiffness ratio, while an increase in the number of panels increases the stiffness ratio only marginally [15, 16].

The ratio S_θ/S_x of torsional strength, S_θ, and lateral strength, S_x, of the idealized systems should also closely match with that of the frame stagings of elevated water tanks for fairly accurate prediction of their inelastic coupled lateral-torsional behaviour. The range of variation of S_θ/S_x ratio for the circular frame-stagings of elevated water tanks are studied elsewhere [18]. The analytical expressions of S_θ, S_x and S_θ/S_x are derived in the same study. It is observed that S_θ/S_x becomes $D/2$ for stagings with lesser number of columns and panels (say, $N_c = 4$ and $N_p = 4$) while for stagings with large number of columns and panels, this ratio becomes D where D represents the diameter of the staging. Hence, the two-element and four-element systems having S_θ/S_x ratio $D/2$ and D, respectively (refer Figure 2), are adequate representative of frame stagings with lesser number of columns and panels, and those with large number of columns and panels respectively.

A staging with large number of columns and panels has higher degree of indeterminacy than that with four columns and small number of panels. Likewise, the idealized four-element system has higher indeterminacy than the idealized two-element system. However, these idealized systems do not represent the exact number of indeterminacy of the elevated water tanks they are modelling. But, behaviour of a staging with four columns with a direction of excitation perpendicular to one of the braces as shown in Figure 2(c) may be adequately featured by two-element model. Let the translation and rotation of staging be u and θ, under coupled lateral–torsional motion. In this case, two columns on one side will have resultant displacement of same magnitude (though the direction of resultant displacement will be different) and will yield together. Similarly, two columns on the other side will have same displacement and should yield together. This possibility of simultaneous yielding of two columns on each side, together, is represented through the concentrated modelling of stiffness in two-element model. The actual rigorous modelling of staging structure is avoided as the objective of the present work is to qualitatively recognize the magnification of inelastic torsional displacement due to strength deterioration, triggered by accidental eccentricity and closely spaced torsional and lateral natural periods. Apart from severe complexity involved, rigorous modelling also may not yield a very accurate result due to the uncertainty involved in other parameters, e.g., actual magnitude and direction of eccentricity, and parameters involved in hysteresis behaviour. The present models with $K_\theta/(K_x D^2)$ and S_θ/S_x ratio conforming to that of real tanks are expected to give at least qualitatively good prediction of staging edge displacement in inelastic range. Such idealized one-storey models are also extensively used to study the seismic behaviour of asymmetric buildings [5].

Following the elevated water tank which suffered a torsional failure in 1993 Killari, India, earthquake [3]; an elevated tank with six columns, four panels of equal heights, staging radius 3·375 m, staging height 18 m, tank container radius 4·45 m and tank container height 4·45 m, is chosen for studying the response. It has column section 460 mm × 460 mm and beam section 200 mm × 500 mm, both made up of M20 grade of concrete. Column sections have 4–20$\overline{\phi}$ and 4–16$\overline{\phi}$ bars as longitudinal reinforcements. Beam sections have 6–20$\overline{\phi}$ bars equally distributed at top and bottom. Both types of members have lateral ties made of 6$\overline{\phi}$ bars at a spacing of 250 mm. This structure is idealized as a four-element system. The actual tank staging is found to have $K_\theta/(K_x D^2) = 0{\cdot}37$ through a finite element analysis. S_θ/S_x for this staging is calculated following the procedure outlined in the literature [18] and found to be around 0·85D. Hence, the idealized system is chosen to have two load-resisting elements, spaced at a distance D apart, in the direction of eccentricity. The other two load-resisting elements in the perpendicular direction have a distance of $0{\cdot}7D$ between them. Due to these locations of load-resisting elements, the idealized system also has $K_\theta/(K_xD^2) = 0{\cdot}37$ and $S_\theta/S_x = 0{\cdot}85$.

2.2. SYSTEM PROPERTIES

In the current study, systems with $T_x = 0{\cdot}5$ s, $1{\cdot}0$ s and $2{\cdot}0$ s, representing the typical natural periods in acceleration-sensitive, velocity- and displacement-sensitive regions, respectively, of the usual design response spectrum, are considered. These values of lateral natural periods also represent the realistic lateral natural periods of elevated water tanks. The natural period ratio $\tau = T_\theta/T_x$ for elevated water tanks generally take values between 0·4 and 1·5 [16]. So, in the present study, τ is varied from 0·25 to 2·0. τ can be expressed as $[r/D]/\sqrt{[K_\theta/(K_xD^2)]}$ where r, the radius of gyration of the mass, may independently change due to the change in container diameter or depth of water in the container, etc. even if D and $K_\theta/(K_xD^2)$ are fixed for a staging.

In this study, small stiffness eccentricity (e) and strength eccentricity ($e_{strength}$) are introduced by changing the stiffness and strengths of lateral load-resisting elements, respectively, without changing the system properties. Small eccentricity cases of both $e_{strength}/D = 0{\cdot}05$, and $e/D = 0{\cdot}05$ are considered.

The idealized four-element system representing the example tank is parametrically adjusted to have $T_x = 1{\cdot}297$ s and $\tau = 0{\cdot}75$. These values are same as those estimated for the actual elevated tank structure represented by the idealized system. The collapsed tank of Killari, India earthquake [3] was having a stair of considerable mass placed eccentrically at one side of the staging. This is found to contribute an eccentricity of approximately $0{\cdot}044D$. For sloshing of water, geometrical imperfection and constructional defects, this may increase further. Thus, for studying the possible effect of lateral–torsional coupling, a small eccentricity of $e/D = 0{\cdot}05$ is introduced into this idealized system.

2.3. EQUATIONS OF MOTION

The equations of motion of the lateral–torsionally coupled systems in the non-linear range can be represented as

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta} \end{Bmatrix} + [C] \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix} + \{p\} = -\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_g(t) \\ 0 \end{Bmatrix}, \tag{2}$$

where u, $\dot{u}$ and $\ddot{u}$ are the lateral displacement, velocity and acceleration of CM with respect to ground, respectively, θ, $\dot{\theta}$ and $\ddot{\theta}$, are the rotational deformation, velocity and acceleration of CM with respect to ground, respectively, $[C]$ is the damping matrix, and $\{p\}$ is the stiffness-related resisting force vector. $[C]$ is chosen such that the damping in each mode of the initial linear elastic system is 2% of the critical damping. The damping matrix so obtained is kept constant throughout the analysis. The stiffness of individual element changes as it undergoes yielding, which introduces non-linearity in $\{p\}$ in the inelastic range.

The non-linear equations of motion (equation (2)) are numerically solved in the time domain by Newmark's γ–β method using the iterative modified Newton–Raphson technique. The Newmark's parameters are chosen as $\gamma = 0{\cdot}5$ and $\beta = 0{\cdot}25$. For systems with lateral natural period, $T_x = 1$ s and 2 s, the time step for integration is taken as 0·01 s, while for systems with $T_x = 0{\cdot}5$ s, it is taken as 0·005 s. These time steps were found to be sufficiently small from sample convergence studies conducted in each case.

3. GROUND MOTIONS USED

A ground motion of 20·48 s duration and consistent with a spectrum similar to the one given in Indian seismic code [19] for 2% of critical damping, is generated by a procedure detailed in literature [20]. The response spectrum regenerated from the synthetic ground motion time history has a maximum departure of around 10% from the target spectrum in the acceleration-sensitive region (i.e., for natural periods upto 0·5 s), as shown in Figure 3(a). The generated time history is shown in Figure 3(b). This ground motion is referred to as spectrum-consistent synthetic ground motion in the rest of the study and is discussed in detail elsewhere [16].

The result of inelastic analysis exhibiting the effect of strength deterioration is presented only under this spectrum-consistent synthetic ground motion to conclude about the broad reflections of the trends in behaviour. While studying the response under four-element system under bi-directional ground motion, another uncorrelated synthetic ground motion of the same duration of 20·48 s is used along the other axis of symmetry of the system. It has a similar response spectrum and same peak ground acceleration as that of the previously mentioned synthetic ground motion.

The fault-parallel and fault-normal pulses generated near strike–slip faults are also employed in the possible simulated idealized forms [as used in the literature (12)] to understand the behaviour of elevated tanks located near faults. Such simulated fault parallel ground motion has a net residual slip; while the fault-normal motion has no residual slip but there is a half-cycle displacement pulse, indicating momentary opening and closing of the earth in slip region. A few recent studies [10–12] caution that near-fault ground motion consisting large duration pulses may result in large deformation demand. The effect of such near-fault ground motions on small-eccentricity systems like elevated water tanks located in fault regions may be crucial in their overall response. Hence, the study of response under such ground motions is also included in the present paper.

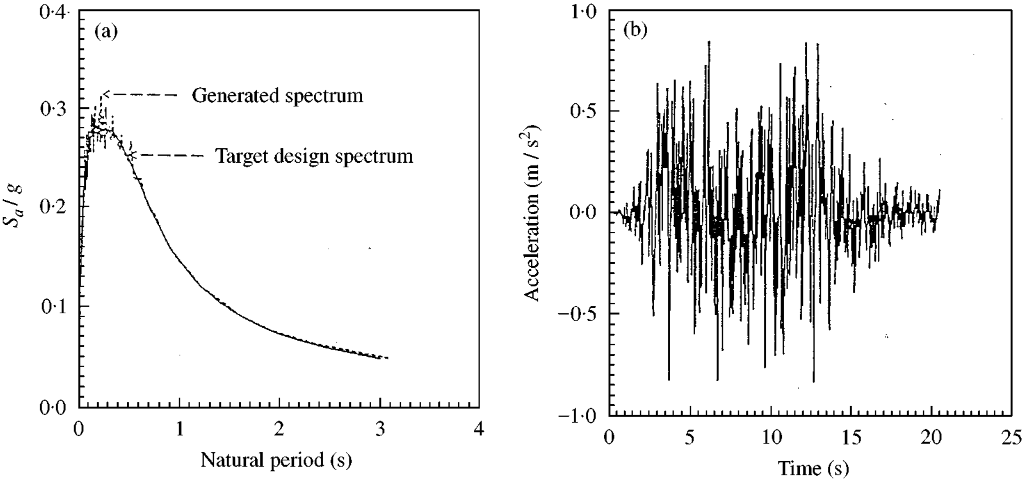

Figure 3. Spectrum-consistent synthetic ground motion used along direction of asymmetry: (a) response spectrum; (b) acceleration–time history.

4. INELASTIC RESPONSE UNDER SPECTRUM-CONSISTENT SYNTHETIC GROUND MOTION

4.1. HYSTERESIS MODEL FOR STRENGTH DETERIORATION

A number of sophisticated hysteresis models are available in the literature [21] for representing the behaviour of reinforced concrete members under cyclic loading, e.g., three-parameter model [22] and Roufaiel–Meyer model [23]. These models incorporate stiffness degradation and pinching characteristics of reinforced concrete members in addition to the strength deterioration characteristics under cyclic loading. But the sophistication of these models can be fully utilized only through the calibration of the parameters through experimental data. However, the objective of the present study is to qualitatively examine the possibility of amplification of ductility demand due to unsymmetric yielding and subsequent progressive strength deterioration in structural elements of small-eccentricity systems like elevated tanks. Hence, the isolated effect of strength deterioration to amplify the ductility demand is studied conveniently through a simple idealized strength deteriorating model employed for each of the load-resisting elements; instead of employing one of the rigorous models incorporating all other characteristics with separate calibration study of its parameters. In this model, the number of yield excursions regulates the extent of strength deterioration. For simplicity, the strength deterioration is considered as a regime-independent phenomenon in this model, i.e., the amount of plastic strain accumulated does not control the extent of strength decay.

A general force–displacement curve demonstrating the hysteresis rules used in the present study is shown in Figure 4, the details of which may be seen elsewhere [16]. The hysteresis rules employed in this simplified strength deteriorating hysteresis model are:

1. The backbone curve is elastic–perfectly plastic.
2. Each yielding on either side, i.e., positive side or negative side, causes a deterioration in the yield force by a definite fraction δ of the original (undeteriorated) yield force. This deterioration is effective only at the next yielding, on either the positive side or the negative side.
3. If a yielding is followed by a small amount of unloading such that the current force after unloading is still higher than the new deteriorated strength after the last yielding, then a further loading will cause immediate yielding at the current force level itself.

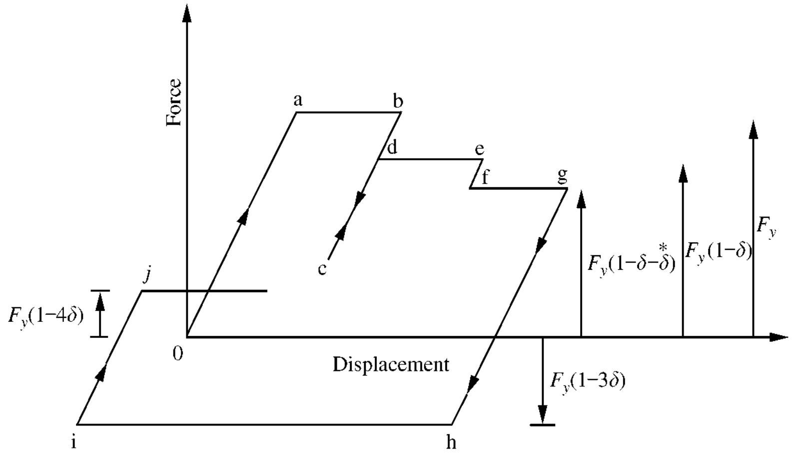

Figure 4. A general force-displacement hysteresis curve based on the proposed simple strength-deteriorating hysteresis model. Note: $\delta^* < \delta$.

Strength deterioration in reinforced concrete members largely depends on their detailing. A properly detailed member may exhibit a very small deterioration in strength, while a poorly detailed member exhibits a very large drop in strength under cyclic loading. However, most of the normally detailed reinforced concrete structures exhibit considerable strength deterioration. Experimental studies are available on the load–deformation behaviour of reinforced concrete members [24–28] which provide load–deformation curves for different reinforced concrete members with different detailing schemes. For the present study, these curves are carefully examined. The total drop in strength is divided by the total number of yield excursions to obtain the average amount of strength deterioration in each yield excursion. From a number of such curves available in the literature [24–28], in most cases, the average rate of deterioration δ in a single yield excursion is found to be around 5% of the initial yield strength for ordinarily detailed reinforced concrete specimen. However, it is found to take values upto 10%. Hence, the values of δ used in this study are 0·0, 0·02, 0·05, 0·08, and 0·1. However, in case of bi-directional ground motion, only cases of $\delta = 0{\cdot}05$ and 0·1 are considered.

4.2. DUCTILITY REDUCTION FACTOR R_μ

The extent of inelasticity in a structure under a specified loading depends on the ratio of the elastic strength demand and the actual lateral strength. The response reduction factor, R, which is the ratio of the maximum lateral strength experienced by the structure if it were to remain elastic and the design lateral force for a system, is one way of indicating the expected extent of inelasticity under a specified loading. Building structures possess large redundancies, and hence, are assigned a large value of R [e.g., NEHRP provisions (29) specify R as high as 8 for certain building systems]. However, the elevated water tank structures do not enjoy such a high degree of redundancy, and are assigned significantly a lower value for R (e.g., $R = 2{\cdot}5$ in NEHRP Recommended Provisions, 1991 [29]). Since factors of safety are involved in the process of design, all structures including elevated water tanks will always possess significant overstrength over and above the design lateral force [30]. This implies that yielding will take place not at the design value of lateral force but at a higher value. A factor called the ductility reduction factor R_μ is defined in the literature as the ratio of the maximum lateral force that will be experienced by the structure if it were to remain elastic and yield lateral force. So, the ductility reduction factor R_μ will be less than the response reduction factor R. This implies that R_μ for elevated water tanks will be less than 2·5. In the current study, two cases of $R_\mu = 1$ and 2 are considered, the former being the elastic case for the symmetric system.

4.3. EFFECT OF STRENGTH DETERIORATION

While presenting results, the maximum element displacement of the eccentric system is normalized with the maximum element displacement of a reference symmetric system with the same lateral natural period T_x, ductility reduction factor R_μ and damping so that the effect of torsion on displacement demand of the load-resisting elements can be clearly visualized.

4.3.1. *Study of two-element systems*

Two-element systems, with both stiffness-eccentric strength-symmetric and stiffness-symmetric strength-eccentric systems, with lateral natural period T_x of 0·5, 1 and 2 s

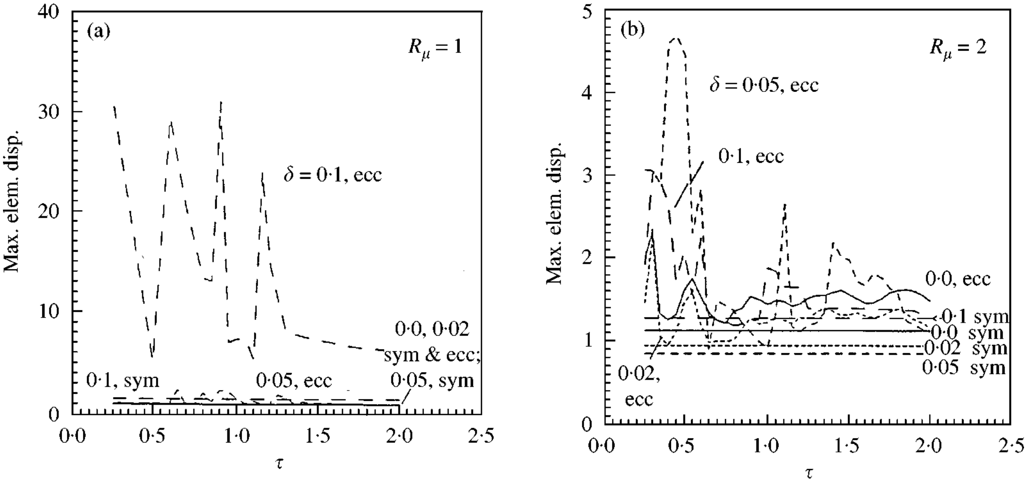

Figure 5. Maximum normalized element displacement of strength-eccentric two-element system ($T_x = 0{\cdot}5$ s, $e_{strength}/D = 0{\cdot}05$).

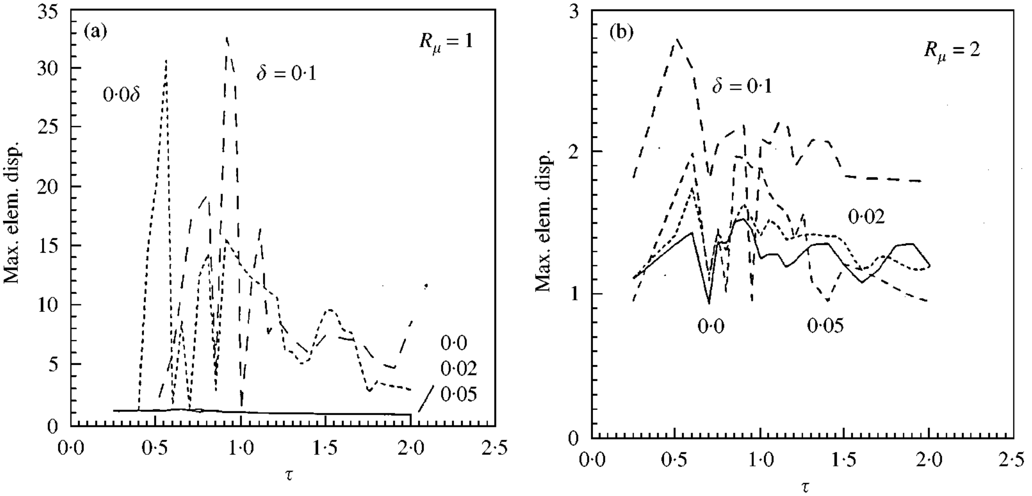

Figure 6. Maximum normalized element displacement of stiffness-eccentric strength-symmetric two-element system ($T_x = 0{\cdot}5$ s, $e/D = 0{\cdot}05$).

have been analyzed. The variation in the normalized maximum element displacement with $\tau(= T_\theta/T_x)$ is studied. Sample responses of stiffness-symmetric strength-eccentric system ($e_{strength}/D = 0{\cdot}05$) and stiffness-eccentric strength-symmetric system ($e/D = 0{\cdot}05$) with lateral natural period $T_x = 0{\cdot}5$ s are presented in Figures 5 and 6 respectively. In each figure, two sets of curves are provided corresponding to the ductility reduction factor $R_\mu = 1$ and 2. These results represent the generalized trends in effect of strength deterioration observed from all other cases studied. So, the results of all other cases, available elsewhere [16] along with the present results, are not included in the present paper. The stiffness-symmetric strength-eccentric systems have overall strength slightly lesser than the reference symmetric system due to slightly smaller strength asymmetrically in one element. Hence, Figure 5 also includes the response of symmetric systems with same lateral overall strength as that of the stiffness-symmetric strength-eccentric systems. The effect of strength eccentricity alone can be identified from this.

For small-eccentricity systems studied in the present paper, the normalized element displacements presented in all the figures are almost same as the ratio of the maximum element ductility demand of these systems normalized with respect to the maximum element ductility demand of the corresponding reference symmetric systems.

Figures 5 and 6 show that the effect of eccentricity on maximum element displacement, and hence, on ductility demand, generally increases with increasing rate of strength deterioration, δ. For a high rate of strength deterioration of $\delta = 0{\cdot}1$ (or 0·08), the eccentric systems show a very high normalized element displacement even for $R_\mu = 1$. The displacement of the symmetric system is small due to an elastic range behaviour when $R_\mu = 1$. As compared to such elastic range displacement, the localized inelastic range displacement of the eccentric systems are found to be very high. The maximum element displacement and ductility demand of eccentric systems are, in many instances, more than twice the element displacement and ductility demand of the reference symmetric systems. A large element displacement is also observed for $R_\mu = 2$ even when $\delta = 0{\cdot}05$ or smaller. Since, a rate of strength deterioration of 0·05 is not unexpected in reinforced concrete members, elevated water tanks can have large displacement and ductility demand in load-resisting elements due to small accidental eccentricity. A small accidental eccentricity causes early yielding of one of the elements. A higher rate of strength deterioration further lowers the strength of that element under repeated loading. This leads to a larger strength eccentricity resulting in a further increase in element displacement, and hence, in the ductility demand. Such a large displacement and ductility capacity is difficult to be provided in reinforced concrete structural members. The effect may be even larger if both stiffness eccentricity and strength eccentricity accidentally occur together.

4.3.2. *Study of four-element systems*

Idealized four-element systems with small stiffness eccentricity ($e/D = 0{\cdot}05$) and strength symmetry are studied to investigate their inelastic behaviour. The variations in normalized maximum element displacement with τ under bi-directional synthetic ground motion for two values of rate of strength deterioration, $\delta = 0{\cdot}05$ and 0·1, and for $R_\mu = 2$, are shown in Figure 7. The responses of the four-element systems under uni-directional ground motion with $\delta = 0{\cdot}1$, are also presented to compare with the effect of bi-directional ground motion. The curves are marked 1-D and 2-D to indicate the responses under uni-directional and bi-directional ground motions respectively. Three values of lateral natural periods, $T_x = 0{\cdot}5$, 1·0 and 2·0 s, are considered. Further details of the same are available elsewhere [16].

As expected, the four-element systems show a greater effect of torsional coupling under bi-directional motion than under uni-directional ground motion. Four-element systems have two additional elements with symmetric characteristics oriented along the perpendicular direction of ground motion in addition to the two elements with asymmetric characteristics oriented along the direction of ground motion. Under uni-directional ground motion, these additional elements may remain elastic. So, even if the stiffness in the direction of ground motion becomes zero because of the yielding of both the elements, the torsional resistance generally may not reduce below 50% of the original torsional stiffness.

However, the situation may be different, if a four-element system is subjected to bi-directional ground motions. In this case, the additional elements are also expected to exhibit considerable post-yield range response during ground shaking owing to the ground motion parallel to their orientation. So, under bi-directional ground motion, the torsional resistance may become zero during ground shaking depending on the correlation between

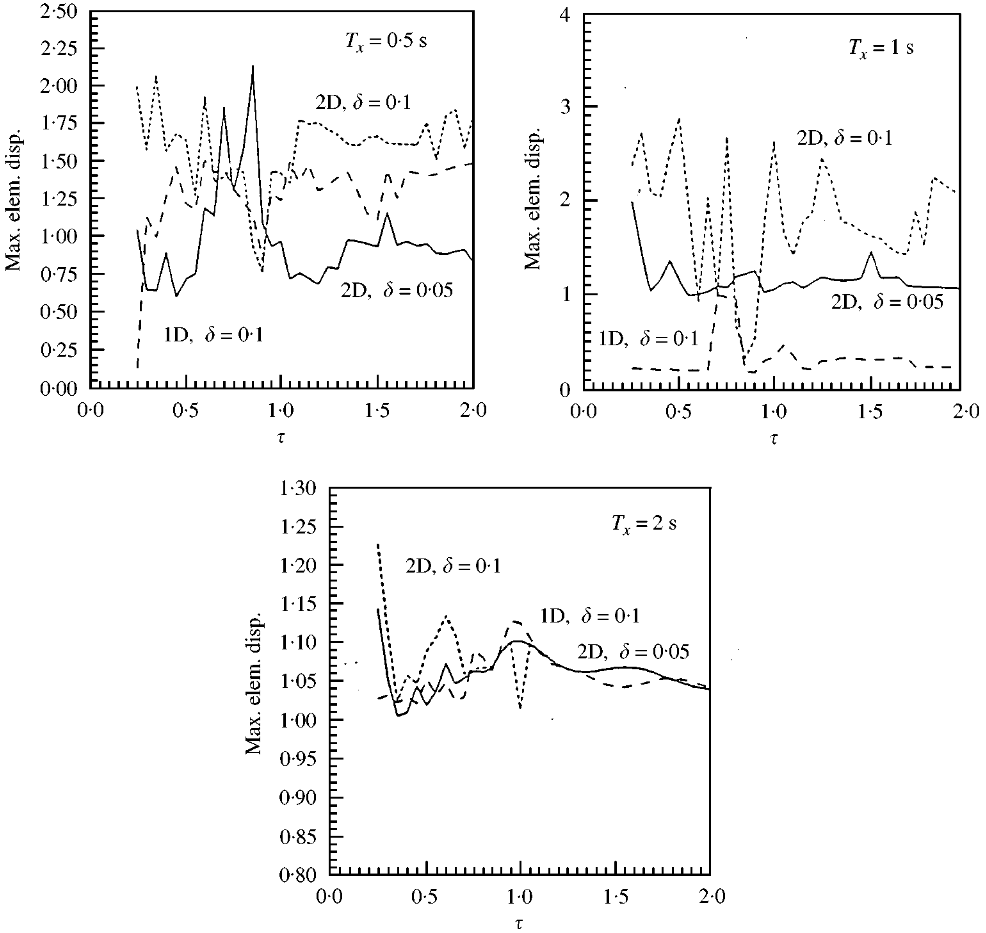

Figure 7. Maximum normalized element displacement of stiffness-eccentric strength-symmetric four-element system under bi-directional and uni-directional ground motions ($R_\mu = 2$).

the ground acceleration in two mutually perpendicular directions. Hence, as observed in an earlier study using an elasto-plastic behaviour [31], the present study also shows that this drop in the torsional stiffness may increase torsional response. Another reason for increasing response under bi-directional ground motion may be the introduction of strength asymmetry in the additional elements due to their unsymmetrical yielding and the subsequent deterioration in strength under the combined action of the lateral motion in the direction of the additional elements (i.e., along the direction of symmetry) and the torsional motion.

Further, if the curves of $\delta = 0{\cdot}1$ and 0·05 in Figure 7 are compared with the corresponding curves for two-element systems for $R_\mu = 2$ in Figure 6, a four-element system exhibits a lesser effect of torsion than two-element systems. This is so because a two-element system has lesser redundancy, torsional stiffness and strength than a four-element system. This implies that the frame stagings with large number of panels and columns represented by four-element systems are less vulnerable to the effect of torsion in the post-yield range.

Under bi-directional ground motion, four-element systems with $T_x = 0{\cdot}5$ and 1 s, indicate high element displacement and ductility demand, upto twice or more than that of the corresponding reference symmetric system; this has serious implications. Such a high displacement and ductility demand caused by torsion may not be acceptable as far as the detailing of lateral load resisting elements is concerned. This clearly indicates that even elevated water tanks supported on staging with many columns and panels are highly vulnerable to the effect of torsional coupling in a real event of earthquake involving bi-directional ground motion.

Comparison of the element displacement curves in Figure 7 for systems with $\delta = 0{\cdot}1$ and 0·05 under bi-directional ground motions shows that the effect of torsional coupling increases with the rate of strength deterioration in four-element systems also.

4.3.3. *Study of example tank through four-element systems*

The idealized four-element system, with $e/D = 0{\cdot}05$ and $\delta = 0{\cdot}1$, is found to have its displacement of one of the load-resisting elements amplified to 2·4 times and 3·9 times that of a similar reference symmetric system for $R_\mu = 1$ and 2 respectively. This indicates the possibility of similar magnification of the staging edge displacement due to small incidental eccentricity and may prove the role of progressive torsional effect in the collapse of the elevated tank during 1993 Killari, India, earthquake [3].

5. INELASTIC BEHAVIOUR OF ELEVATED TANKS LOCATED NEAR TECTONIC FAULTS

The inelastic torsional behaviour of large-eccentricity systems with $e/r = 0{\cdot}5$ (appropriate for buildings) under fault-normal ground motion was reported to be significant [11]. Inelastic behaviour of steel planar building frames under fault-parallel and fault-normal ground motions has also been reported to be very critical [12]. A similar study on elevated water tanks supported on frame stagings is expected to provide crucial inputs in their design.

5.1. VARIATION OF PARAMETERS

The natural period ratio τ is varied from 0·25 to 2. Four values of T_x/T_1, namely 0·05, 0·5, 1·0 and 5, are considered where T_1 denotes the duration of near-fault pulses. Two sets of yielding systems with ductility reduction factor $R_\mu = 1$ and 2 are considered. For all systems studied, eccentricity is taken as $e/D = 0{\cdot}05$. Stiffness-eccentric strength-symmetric two-element systems alone are considered. Elasto-plastic behaviour is assumed for each lateral load-resisting elements. Hence, the same hysteresis model with $\delta = 0{\cdot}0$ is used for the study.

5.2. RESULTS AND DISCUSSION

Again, the maximum element displacement of the eccentric system is normalized with respect to the displacement of the corresponding symmetric system. The variation in this normalized maximum element displacement with τ, for different values of T_x/T_1, under fault-parallel ground motion, is presented in Figure 8. Similarly, the variation under fault-normal ground motion is presented in Figure 9.

Figures 8 and 9 show that, for small-eccentricity systems ($e/D = 0{\cdot}05$), the torsional coupling effect increases the maximum element displacement and hence, ductility by only a small amount (around 20–25%) as compared to a similar symmetric system.

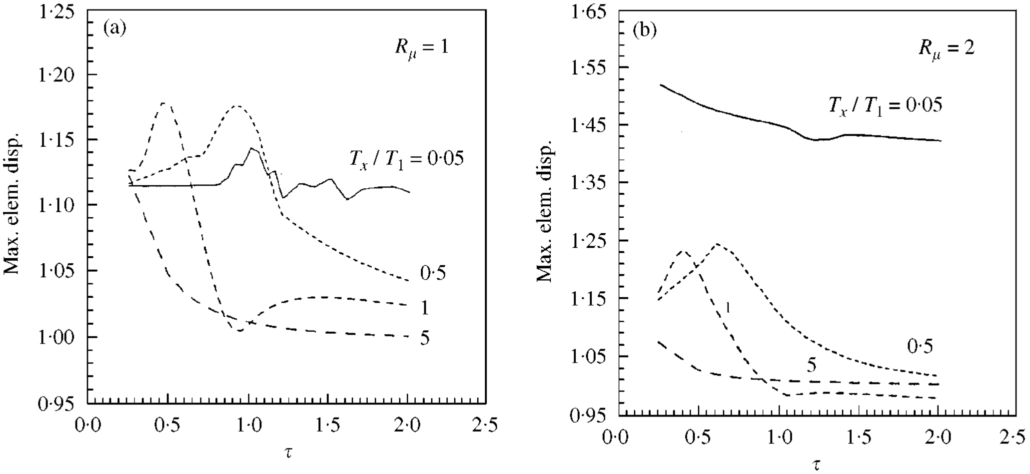

Figure 8. Maximum normalized element displacement of stiffness-eccentric two-element system under fault-parallel ground motion ($e/D = 0·05$).

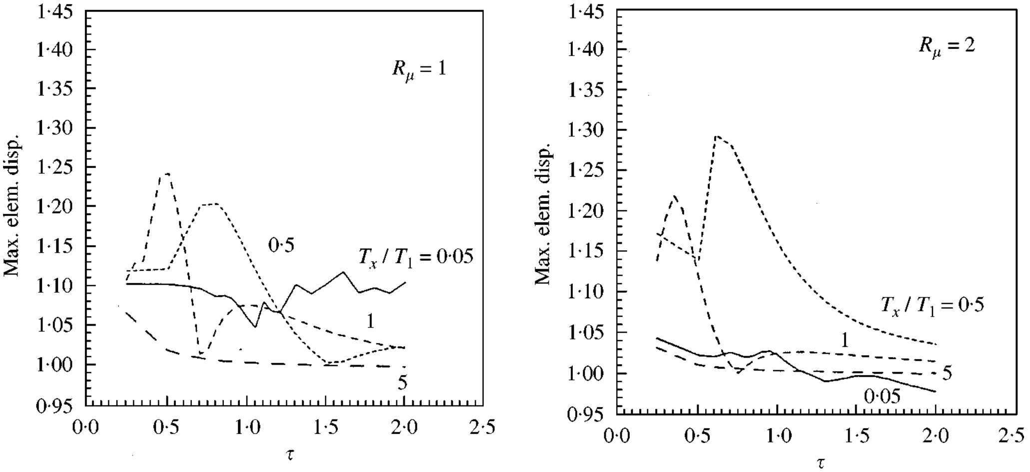

Figure 9. Maximum normalized element displacement of stiffness-eccentric two-element system under fault-normal ground motion ($e/D = 0·05$).

The variation of element displacement ductility demand of a symmetric system for $R_\mu = 2$ with T_x/T_1 varying from 0·2 to 5 is shown in Figure 10 (more details are available in [16]). This figure shows that when the pulse duration T_1, is large, i.e., T_x/T_1 is small, the symmetric system itself produces a very high ductility demand. Ductility demand stabilizes at a value of around 2, when the pulse duration becomes less than the lateral natural period (i.e., $T_x/T_1 \geqslant 1$). So, a pulse of large duration may produce a large ductility demand in a system due to the increased amplitude in lateral translation irrespective of symmetry or small eccentricity in it. Since, a pulse duration of around 2 s is not unexpected, elevated water tanks having lateral natural period less than 2 s may encounter a large ductility demand if situated near a tectonic fault. Large displacement may not be acceptable in elevated water tanks even from the operational point of view, e.g., fracture of the connected pipelines. Moreover, such structures having large masses concentrated at considerable heights may undergo collapse owing to secondary P–Δ effects.

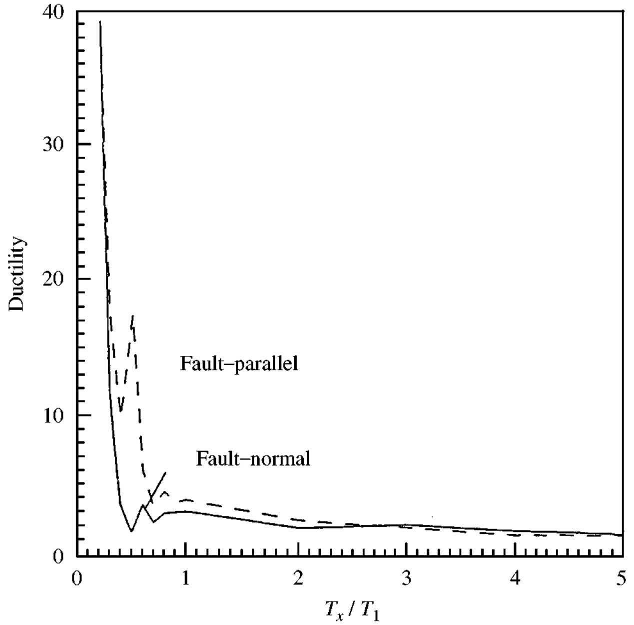

Figure 10. Variation of element displacement ductility demand in stiffness and strength symmetric systems under near-fault ground motions with different pulse durations.

6. CONCLUSIONS

In the context of torsional failure of elevated tanks in earthquakes, present paper aims to study the torsional behaviour of water tanks supported on concrete frame-type staging (Figure 1), using simple idealized models. The salient conclusions of the study are listed below.

1. Small accidental eccentricity may cause asymmetrically localized yielding in the staging members due to unequal displacement of staging edges caused by coupled lateral–torsional vibration [9]. The progressive strength drop, due to strength-deteriorating characteristics of reinforced concrete members, in these localized regions causes continuous shifting of strength centres increasing the strength eccentricity progressively. This behaviour is found to increase the effect of dynamic torsional response significantly and generate a high localized displacement and ductility demand in staging load-resisting elements. These demands are much larger than the displacement demand expected from a perfectly symmetrical system. The effect becomes more severe with increase in rate of strength deterioration of concrete. Such effect of torsional coupling is found to be present not only in systems designed to behave inelastically ($R_\mu = 2$) but also in systems marginally designed to behave elastically without any overstrength ($R_\mu = 1$).

This large displacement and ductility demand cannot be accommodated in reinforced concrete members. Hence, this may result in a collapse due to torsion as observed in the elevated tank collapsed in 1993 Killari, India, earthquake [3]. The strength deterioration characteristic is found to be dependent on quality control and primarily on reinforcement detailing of concrete. But unless any definite acceptable quantitative guideline of such dependence is found out, the phenomenon of strength deterioration and hence, progressive localized torsional damage cannot be eliminated. Thus, to avoid the possibility of failure due to torsional coupling, such structures should be designed with adequate overstrength to behave elastically under the critical design earthquake.

2. Between two similar elevated tanks, (i.e., same lateral time period, T_x, the time period ratio, τ, normalized eccentricity, damping and diameter D), the tank supported on staging with less number of columns and panels (represented by two-element systems) are found to be torsionally more severely vulnerable compared to the tank supported on staging with large number of columns and panels (represented by four-element systems). Such behaviour may be attributed to the lesser torsional-to-lateral stiffness ratio and lesser torsional-to-lateral strength ratio of the former as compared to the latter. So, staging with small number of columns and panels should be avoided as far as possible.
3. The elevated tanks near faults may also demand very high displacement and ductility capacity in the inelastic range under a long duration pulse, characteristics of near-fault motion, due to a purely translational behaviour; though the torsional effect may increase the displacement demand of the staging members by only around 20–25%. Such large displacement demand cannot be allowed due to operational requirement and possibility of failure owing to secondary P–Δ effects. So, these tanks should also be designed to exhibit elastic response under design earthquake.

The existing studies listed in the literature [4, 5] on inelastic torsional behaviour employs bilinear or stiffness-degrading hysteresis rules to model the behaviour of structural elements. In light of the considerable increase in element displacement and ductility demand due to the effect of strength deterioration, the applicability of the results of these studies becomes questionable for asymmetric reinforced concrete structures. The effect of strength deterioration must be included in the hysteresis behaviour of load-resisting elements to study the inelastic torsional behaviour of these structures. However, the conclusions of the present study should also be extensively verified through many more case studies, due to the possibility of variation of results in inelastic dynamic analyses for small variation in parameters.

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